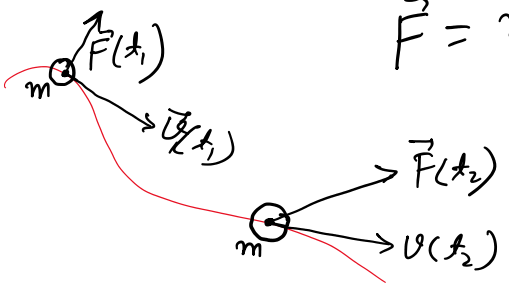
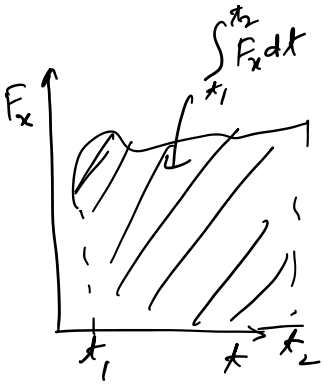
Impulse - Momentum Method

Thursday, March 27, 2025 5:53 PM

$$\vec{F} = m\vec{a}$$





$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\int_{t_1}^{t_2} \vec{F}\,dt = \int_{\vec{v}_1}^{\vec{v}_2} m\,d\vec{v}$$

$$= m\vec{v}_2 - m\vec{v}_1$$

$m\vec{v} = \vec{p}$ = Linear momentum of a particle of mass m

$\int_{t_1}^{t_2} \vec{F}\,dt$ = Linear Impulse of a Force $\vec{F}(t)$ between t_1 and t_2

Impulse - Momentum principle

$$\int_{t_1}^{t_2} \vec{F}(t)\,dt = \vec{p}(t_2) - \vec{p}(t_1)$$

$$\vec{p}(t_1) + \int_{t_1}^{t_2} \vec{F}(t)\,dt = \vec{p}(t_2)$$

$$\vec{F}(t) = \dot{\vec{p}}$$

System of particles

m_G = Const.

$$\vec{F}_i + \sum_{\substack{j=1 \\ j\neq i}}^{N} \vec{f}_{ij} = \dot{\vec{p}}_i$$

$$\sum \vec{F}_i + \sum_i \sum_{\substack{j \\ j\neq i}} \vec{f}_{ij} = \sum_i \dot{\vec{p}}_i$$

$$\vec{F} = \dot{\vec{p}}$$

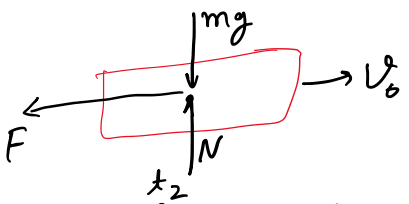
$\vec{F}$ is zero

$$\dot{\vec{p}} = 0 \Rightarrow \boxed{\vec{p}(t_1) = \vec{p}(t_2)}$$

... system of linear

$\vec{p} = 0 \Rightarrow \boxed{p(t_1) \ldots}$

Conservation of Linear Momentum.

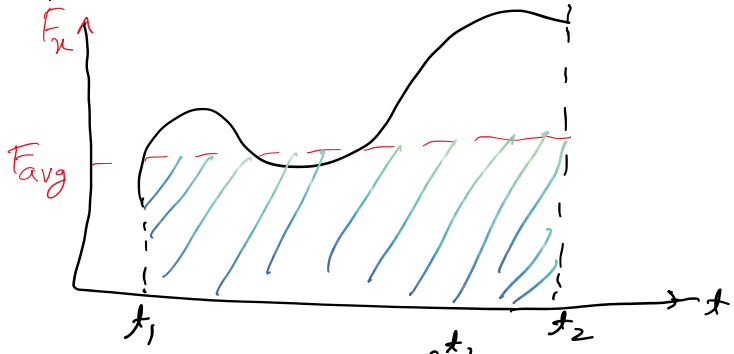


$v_x(t_1) = v_0$ | $v_x(t_2) = 0$

$v_y(t_1) = 0$ | $v_y(t_2) = 0$

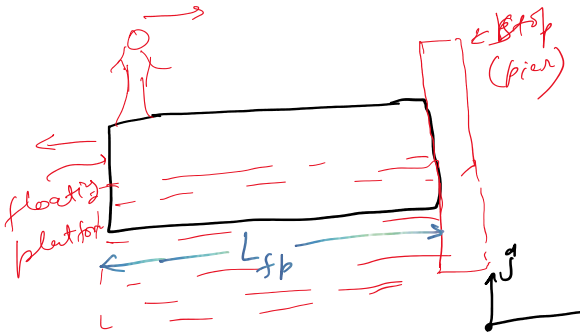
$$\int_{t_1}^{t_2} F_x\,dt = p_x(t_2) - p_x(t_1) = m\cancel{v_x(t_2)}^{0} - m v_x(t_1)$$
$$= -m v_x(t_1) = -m v_0$$

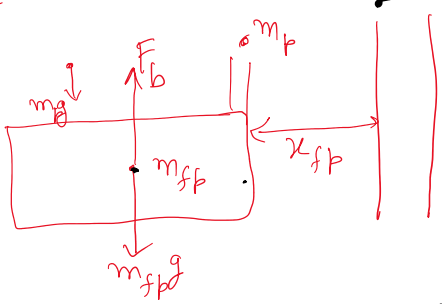
$$\int_{t_1}^{t_2} F_y\,dt = p_y(t_2) - p_y(t_1)$$
$$= 0 \quad \Rightarrow F_y = 0$$



$$F_{avg_x} \times (t_2 - t_1) = \int_{t_1}^{t_2} F_x(t)\,dt$$
$$= p_2 - p_1$$

$$F_{avg_x} = \frac{p_2 - p_1}{t_2 - t_1}$$





$x_{p_1} = -L_{fp}$

$x_{fp_1} = 0$

$$(m_p + m_{fp})\, v_{G_x}(t_1) = (m_p + m_{fp})\, v_{G_x}(t)$$

m_{fb}

$$(m_b + m_{fb})\, v_{G_x}(t_1) = (m_b + m_{fb})\, v_{G_x}(t)$$

$$v_{G_x}(t_1) = v_{G_x}(t) = 0$$

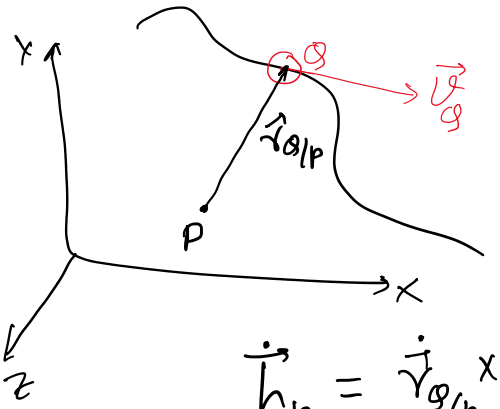
$$\dot{x}_G = 0$$

$$x_G(t_1) = x_G(t_2)$$

$$\frac{m_b x_{b_1} + m_{fb} x_{fb_1}}{m_b + m_{fb}} = \frac{m_b x_{b_2} + m_{fb} x_{fb_2}}{m_b + m_{fb}}$$

$$\boxed{x_{fb_2} = \left(\frac{m_b}{m_b + m_{fb}}\right) L_{fb}}$$

Angular Momentum



Linear momentum

$$\vec{P}_G = m\vec{v}_G$$

Angular momentum

$$\vec{h}_P = \vec{r}_{G/P} \times \vec{P}_G$$

$$\dot{\vec{h}}_P = \dot{\vec{r}}_{G/P} \times m\vec{v}_G + \vec{r}_{G/P} \times m\vec{a}_G$$

$$= (\vec{v}_G - \vec{v}_P) \times m\vec{v}_G + \vec{r}_{G/P} \times \vec{F}_G$$

$$\vec{M}_P = \dot{\vec{h}}_P + \vec{v}_P \times m\vec{v}_G$$

$$\boxed{\vec{M}_P = \dot{\vec{h}}_P}$$

$$\int_{t_1}^{t_2} \vec{M}_P\, dt = \vec{h}_P(t_2) - \vec{h}_P(t_1)$$

$$\boxed{h_P(t_1) + \int_{t_1}^{t_2} \vec{M}_P\, dt = \vec{h}_P(t_2)}$$

$$h_p(t_1) + \int_{t_1}^{t_2} M_p \, dt = h_p(t_2)$$

Angular Impulse-Momentum principle

$\vec{M}_p = 0$

$\vec{h}_p(t_1) = \vec{h}_p(t_2)$

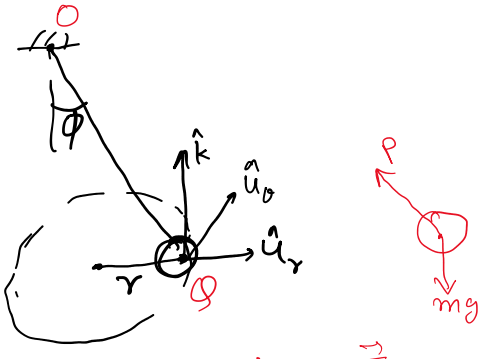
System of Particles

$$\vec{M}_{P_i} = \dot{\vec{h}}_{P_i} + \vec{v}_P \times m_i \vec{v}_i$$

$$\sum_{i=1}^{N} \vec{M}_{P_i} = \sum_{i=1}^{N} \dot{\vec{h}}_{P_i} + \vec{v}_P \times \sum_{i=1}^{N} m_i \vec{v}_i$$

$$\vec{M}_P = \dot{\vec{h}}_P + \vec{v}_P \times m_G \vec{v}_G$$

$$\vec{M}_P = \dot{\vec{h}}_P$$



$$\vec{M}_O = \vec{r}_{Q/O} \times \vec{F}_Q$$

$$= L(\sin\phi \hat{u}_r - \cos\phi \hat{k}) \times [-P(\sin\phi \hat{u}_r - \cos\phi \hat{k}) - mg\hat{k}]$$

$$= mgL\sin\phi \hat{u}_\theta$$

$$\vec{h}_O = h_{O_r} \hat{u}_r + h_{O_\theta} \hat{u}_\theta + h_{O_k} \hat{k}$$

$$\dot{\vec{h}}_O = \dot{h}_{O_r} \hat{u}_r + h_{O_r} \dot{\hat{u}}_r + \dot{h}_{O_\theta} \hat{u}_\theta + h_{O_\theta} \dot{\hat{u}}_\theta + \dot{h}_{O_k} \hat{k}$$

$$mgL\sin\phi \hat{u}_\theta = \vec{M}_O = (\dot{h}_{O_r} - \dot{\theta} h_{O_\theta}) \hat{u}_r + (\dot{h}_{O_\theta} + \dot{\theta} h_{O_r}) \hat{u}_\theta + \dot{h}_{O_k} \hat{k}$$

$$\dot{h}_{O_k} = 0$$

$$h_{O_k} = h_{O_z}$$

$$h_{O_k}$$

$$h_{O_{k_1}} = h_{O_{k_2}}$$

$$\dot{h}_{O_\gamma} - \dot{\theta}\, h_{O_\beta} = 0$$

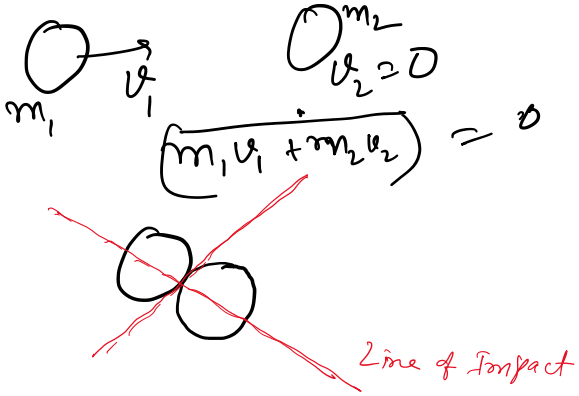
$$\vec{F} = m_G \vec{a}_G = \dot{\vec{p}}_G$$

$$\vec{M}_P = \dot{\vec{h}}_P + \vec{v}_P \times m\vec{v}_G$$

Euler's First and second Law

Impact

- short Dynamic Events
- Can be considered Instantaneous
- velocity can be finite



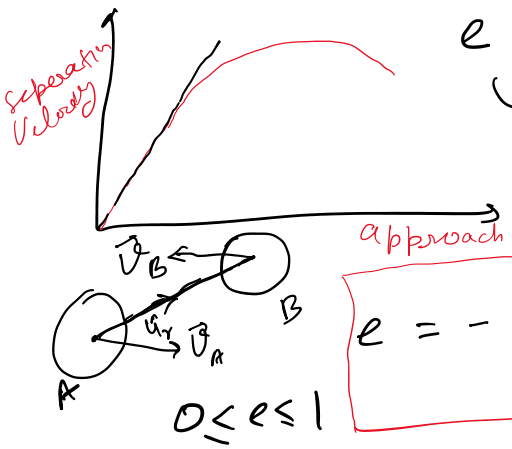
$\Delta t = t_2 - t_1 \to 0$

Impulsive Force → ∆ change of momentum is finite over an infinitesimal time interval

$$\vec{F}_{avg} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} \to \infty$$

$v_A > v_B$

$$m_A v_A(t^-) + m_B v_B(t^-) = m_A v_A(t^+) + m_B v_B(t^+)$$



e = Separation velocity / Approach velocity

↳ Coefficient of Restitution

$$e = -\frac{(\vec{v}_a^{+} - \vec{v}_b^{+}) \cdot \hat{u}_y}{(\vec{v}_a^{-} - \vec{v}_b^{-}) \cdot \hat{u}_y}$$

$0 \leq e \leq 1$